Zadania

1. Obliczyć pochodne funkcji:

a) $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^4 + \frac{13}{5}x^5 - 2x^6, \quad g(x) = 5x^{15} - x^2 + \frac{1}{3}x - 2$

b) $f(x) = 3x^{\frac{7}{3}} - 14x^{\frac{13}{4}} + \frac{4}{7}x^{-\frac{1}{2}} + 7^{\frac{3}{2}}, \quad g(x) = \dfrac{2 - x^2}{\sqrt{2 + \sqrt{3}}}$

c) $f(x) = 4x\sqrt[3]{x}, \quad g(x) = \frac{4}{x^3}$

d) $f(x) = \sqrt[5]{x^2}, \quad g(x) = 5\sqrt[3]{x^7}$

e) $f(x) = x^3 \cos x, \quad g(x) = \sin x \cos x, \quad h(x) = x \ln x$

f) $f(x) = (5x^7 - 2x^3 + x - 20)e^x, \quad g(x) = (9x^7 + 3x^{-5} - 3x^{-11})2^x, \quad h(x) = x \arcsin x$

g) $f(x) = \dfrac{4x^7 + 3x^5 - 2x^4 + 7x - 2}{3x^4}, \quad g(x) = \dfrac{2 - x^2}{2x^3 + x + 3}, \quad h(x) = \dfrac{8x^3}{x^3 + x - 1}$

h) $f(x) = \dfrac{3x^2}{7x^5 - x + 2}, \quad g(x) = \dfrac{x^2 - 2x + 3}{x^2 + 2x - 3}, \quad h(x) = \dfrac{5}{2x^2 - 5x + 1}$

i) $f(x) = \sqrt{3x^2 - 7x + 12}, \quad g(x) = (4x^5 - 7x^3 + 14x^2 - 5)^3$

j) $y = \sin 4x, \quad s = (3t + 1)^7, \quad v = (4z^2 - 5z + 13)^{\frac{1}{3}}$

k) $f(x) = \cos^3 x, \quad g(x) = \operatorname{tg}^4 x, \quad h(x) = \arcsin \frac{2}{x}$

l) $f(x) = e^{-x}, \quad g(x) = e^{4x^3 - 6x + 1}, \quad h(x) = e^{\sin x}$

m) $f(x) = 5^x + 2^x, \quad g(x) = 2 \cdot 7^x - 1, \quad h(x) = 4^x - x^2 + 16$

n) $f(x) = 7 \cdot 5^{10x}, \quad g(x) = 5 \cdot 10^{3x}, \quad h(x) = 3^x \cdot x^3$

o) $f(x) = 10 \ln x + e^x, \quad g(x) = \ln 3x, \quad h(x) = 5 \ln 10x$

p) $f(x) = 3 \ln \frac{5}{x-2}, \quad g(x) = \ln \sin x, \quad h(x) = \log_3 x$

r) $f(x) = \log_5 (x^2 - 1), \quad g(x) = \log_4(36 - x^2), \quad h(x) = \sqrt{\ln x}$

s) $f(x) = x^x, \quad g(x) = x^{\sin x}, \quad h(x) = x^{5x}$, dla $x > 0$

2. Obliczyć pochodne:

a) $\left[\ln\left(\dfrac{x^2 + 2}{x^4 + 7}\right)\right]' =$

b) $\left[\left(\dfrac{x^2 + 2x + 1}{x^3 + 27}\right)^3\right]' =$

c) $\left[\left(3 + \dfrac{1}{x^2}\right) \cdot \ln\sqrt{x} + e^3\right]' =$

d) $\left[\left(\sqrt{x} + \dfrac{2}{x}\right) \cdot \ln(x^2 + 4) + \sqrt[3]{2}\right]' =$

e) $\left[\left(6\sqrt[3]{x} - \dfrac{3}{x^3}\right) \cdot e^{x^2 + 5x + 4} + \sqrt{2}\right]' =$

f) $\left[\left(3x^2 - \dfrac{1}{x} + \sqrt{x}\right) \cdot e^{\sin x + 5} + e^3\right]' =$

g) $\left(\sqrt{\ln\left[\arcsin\frac{x^2+5x+7}{x^2+1}\right]}\right)' =$

h) $\left(\sqrt[3]{1+\frac{5}{\operatorname{tg} x}}\cdot \operatorname{arc\,tg}^2\frac{1}{x}\right)' =$

i) $\left(e^{\sin^3 x+7}\cdot\cos\sqrt{x^2+1}+\sqrt{7}\right)' =$

j) $\left(\sin\left[\ln\sqrt{\frac{x^3+4x^2+1}{x^2+2}}\right]\right)' =$

k) $\left(\operatorname{arc\,tg}\sqrt[3]{\operatorname{tg} x}\cdot e^{\arcsin\left(\frac{1}{x}\right)}+\sqrt{2}\right)' =$

l) $\left(\ln\left[\frac{\cos^2 x+1}{\sin x}\right]\cdot\cos\sqrt{\frac{3}{x}}\right)' =$

m) $\left(e^{\sqrt{\arcsin\frac{1}{x}}}\cdot\ln\frac{\sin(4x^2+5x+2)}{\operatorname{ctg} x}\right)' =$

n) $\left(\left[\cos^3(\operatorname{arctg}\sqrt[5]{x^7})\cdot\operatorname{tg}\frac{3}{x^4}+\sqrt{2}\right]^3\right)' =$

o) $\left(\left(\operatorname{arctg}\frac{1}{x}\right)^{\ln(2x)}\right)' =$

p) $\left(\operatorname{tg}^4(\cos 7x)\cdot\sin\frac{\operatorname{arctg}(e^x)}{x^3}+\sqrt{3}\right)' =$

q) $\left(\ln\left[\sqrt{\arcsin(e^{\sqrt[3]{x^2}})}\cdot\operatorname{ctg}(\ln x)\right]\right)' =$

r) $\left(\left(\arcsin\frac{3}{x^3}\right)^{\cos\sqrt[3]{x}}\right)' =$

Odpowiedzi

1. a) $f'(x)=x^2-6x^3+13x^4-12x^5,\quad g'(x)=75x^{14}-2x+\frac{1}{3}$

   b) $f'(x)=7x^{\frac{4}{3}}-\frac{91}{2}x^{\frac{9}{4}}-\frac{2}{7}x^{-\frac{3}{2}}, x\neq 0\quad g'(x)=\dfrac{-2x}{\sqrt{2+\sqrt{3}}}$

   c) $f'(x)=\frac{13}{3}\sqrt[3]{x},\quad g'(x)=-\frac{12}{x^4}, x\neq 0$

   d) $f'(x)=\dfrac{2}{5\sqrt[5]{x^3}}, x\neq 0\quad g'(x)=\dfrac{35}{3}\sqrt[3]{x^4}$

   e) $f'(x)=3x^2\cos x-x^3\sin x,\quad g'(x)=\cos^2 x-\sin^2 x,\quad h'(x)=\ln x+1, x>0$

   f) $f'(x)=(5x^7+35x^6-2x^3-6x^2+x-19)e^x,$
   $g'(x)=2^x\left[(9x^7+3x^{-5}-3x^{-11})\ln 2+63x^6-15x^{-6}+33x^{-12}\right], x\neq 0$
   $h'(x)=\arcsin x+\dfrac{x}{\sqrt{1-x^2}}, x\in(-1,1)$

   g) $f'(x)=\dfrac{12x^7+3x^5-21x+8}{3x^5}, x\neq 0$
   $g'(x)=\dfrac{2x^4-13x^2-6x-2}{(2x^3+x+3)^2}, 2x^3+x+3\neq 0$
   $h'(x)=\dfrac{16x^3-24x^2}{(x^3+x-1)^2}, x^3+x-1\neq 0$

h) $f'(x) = \dfrac{-63x^5 - 3x^2 + 12x}{(7x^5 - x + 2)^2}, 7x^5 - x + 2 \neq 0$

$g'(x) = \dfrac{4x^2 - 12x}{(x^2 + 2x - 3)^2}, x \neq -3 \wedge x \neq 1$

$h'(x) = \dfrac{-20x + 25}{(2x^2 - 5x + 1)^2}, 2x^2 - 5x + 1 \neq 0$

i) $f'(x) = \dfrac{6x - 7}{2\sqrt{3x^2 - 7x + 12}}$

$g'(x) = 3(4x^5 - 7x^3 + 14x^2 - 5)^2(20x^4 - 21x^2 + 28x)$

j) $y' = 4\cos 4x, \quad s' = 21(3t+1)^6, \quad v' = \dfrac{8z - 5}{3\sqrt[3]{(4z^2 - 5z + 13)^2}}$

k) $f'(x) = -3\cos^2 x \sin x$

$g'(x) = 4\operatorname{tg}^3 x \cdot \frac{1}{\cos^2 x} = \dfrac{4\sin^3 x}{\cos^5 x}, x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$h'(x) = \dfrac{-2}{z\sqrt{1 - \frac{4}{x^2}}}, x \in (-\infty, -2) \cup (2, +\infty)$

l) $f'(x) = -e^{-x}, \quad g'(x) = (12x^2 - 6)e^{4x^3 - 6x + 1}, \quad h'(x) = \cos x \cdot e^{\sin x}$

m) $f'(x) = 5^x \cdot \ln 5 + 2^x \cdot \ln 2, \quad g'(x) = 2\ln 7 \cdot 7^x, \quad h'(x) = \ln 4 \cdot 4^x - 2x$

n) $f'(x) = 70\ln 5 \cdot 5^{10x}, \quad g'(x) = 15\ln 10 \cdot 10^{3x}, \quad h'(x) = 3^x \cdot x^2(x\ln 3 + 3)$

o) $f'(x) = \dfrac{10}{x} + e^x, x \neq 0 \quad g'(x) = \dfrac{1}{x}, x \neq 0, \quad h'(x) = \dfrac{5}{x}, x \neq 0$

p) $f'(x) = \dfrac{-3}{x - 2}, x \neq 2, \quad g'(x) = ctgx, \quad h'(x) = \dfrac{1}{x\ln 3}, x \neq 0$

r) $f'(x) = \dfrac{2x}{\ln 5(x^2 - 1)}, x \neq -1 \wedge x \neq 1$

$g'(x) = \dfrac{-2x}{\ln 4(36 - x^2)}, x \neq -6 \wedge x \neq 6$

$h'(x) = \dfrac{1}{2x\sqrt{\ln x}}, x > 1$

s) $f'(x) = x^x(\ln x + 1), x > 0$

$g'(x) = x^{\sin x}\left(\dfrac{\sin x}{x} + \cos x \ln x\right), x > 0$

$h'(x) = 5x^{5x}(\ln x + 1), x > 0$

2. a) $\dfrac{-2x^5 - 8x^3 + 14x}{(x^4 + 7)(x^2 + 2)}$

b) $3\left(\dfrac{x^2 + 2x + 1}{x^3 + 27}\right) \cdot \dfrac{-x^4 - 4x^3 - 3x^2 + 54x + 54}{(x^3 + 27)^2}$

c) $-\dfrac{2}{x^3} \cdot \ln\sqrt{x} + \left(3 + \dfrac{1}{x^2}\right) \cdot \dfrac{1}{2x}$

d) $\left(\dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^2}\right) \cdot \ln(x^2 + 4) + \left(\sqrt{x} + \dfrac{2}{x}\right) \cdot \dfrac{2x}{x^2 + 4}$

e) $\left(\dfrac{2}{\sqrt[3]{x^2}} + \dfrac{9}{x^4}\right) \cdot e^{x^2 + 5x + 4} + \left(6\sqrt[3]{x} - \dfrac{3}{x^3}\right) e^{x^2 + 5x + 4} \cdot (2x + 5)$

f) $\left(6x+\frac{1}{x^2}+\frac{1}{2\sqrt{x}}\right)\cdot e^{\sin x+5}+\left(3x^2-\frac{1}{x}+\sqrt{x}\right)\cdot e^{\sin x+5}\cdot\cos x$

g) $\frac{1}{2\sqrt{\ln\left(\arcsin\frac{x^2+5x+7}{x^2+1}\right)}}\cdot\frac{1}{\arcsin\frac{x^2+5x+7}{x^2+1}}\cdot\frac{1}{\sqrt{1-\left(\frac{x^2+5x+7}{x^2+1}\right)^2}}$
$\cdot\frac{(2x+5)(x^2+1)-2x(x^2+5x+7)}{(x^2+1)^2}$

h) $\frac{1}{3\sqrt[3]{\left(1+\frac{5}{\operatorname{tg}x}\right)^2}}\cdot\frac{-5}{\operatorname{tg}^2 x}\cdot\frac{1}{\cos^2 x}\cdot\operatorname{arc\,tg}^2\frac{1}{x}+\sqrt[3]{1+\frac{5}{\operatorname{tg}x}}\cdot 2\operatorname{arc\,tg}\frac{1}{x}\cdot\frac{1}{1+\left(\frac{1}{x}\right)^2}\cdot$
$\left(-\frac{1}{x^2}\right)$

i) $e^{\sin^3 x+7}\cdot(3\sin^2 x\cos x)\cdot\cos\sqrt{x^2+1}+e^{\sin^3 x+7}\cdot(-\sin\sqrt{x^2+1})\cdot\frac{2x}{2\sqrt{x^2+1}}$

j) $\cos\left(\ln\sqrt{\frac{x^3+4x^2+1}{x^2+2}}\right)\cdot\frac{1}{\sqrt{\frac{x^3+4x^2+1}{x^2+2}}}\cdot\frac{1}{2\sqrt{\frac{x^3+4x^2+1}{x^2+2}}}$
$\cdot\frac{(3x^2+8x)(x^2+2)-2x(x^3+4x^2+1)}{(x^2+2)^2}$

k) $\frac{1}{1+(\sqrt[3]{\operatorname{tg}x})^2}\cdot\frac{1}{3\sqrt[3]{\operatorname{tg}^2 x}}\cdot\frac{1}{\cos^2 x}\cdot e^{\arcsin\frac{1}{x}}$
$+\operatorname{arc\,tg}(\sqrt[3]{\operatorname{tg}x})\cdot e^{\arcsin\frac{1}{x}}\cdot\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\cdot\left(-\frac{1}{x^2}\right)$

l) $\frac{\sin x}{\cos^2 x+1}\cdot\frac{-2\cos x\sin^2 x-\cos x(\cos^2 x+1)}{\sin^2 x}\cdot\cos\sqrt{\frac{3}{x}}$
$+\ln\left(\frac{\cos^2 x+1}{\sin x}\right)\cdot\left(-\sin\sqrt{\frac{3}{x}}\right)\cdot\frac{1}{2\sqrt{\frac{3}{x}}}\cdot\left(-\frac{3}{x^2}\right)$

m) $e^{\sqrt{\arcsin\frac{1}{x}}}\cdot\frac{1}{2\sqrt{\arcsin\frac{1}{x}}}\cdot\frac{1}{\sqrt{1-\frac{1}{x^2}}}\cdot\ln\frac{\sin(4x^2+5x+2)}{\operatorname{ctg}x}$
$+e^{\sqrt{\arcsin\frac{1}{x}}}\cdot\frac{\operatorname{ctg}x}{\sin(4x^2+5x+2)}\cdot\frac{(8x+5)\cos(4x^2+5x+2)\cdot\operatorname{ctg}x-\sin(4x^2+5x+2)\cdot\frac{-1}{\sin^2 x}}{\operatorname{ctg}^2 x}$

n) $3\left(\cos^3(\operatorname{arctg}\sqrt[5]{x^7})\cdot\operatorname{tg}\frac{3}{x^4}+\sqrt{2}\right)^2\cdot\left(3\cos^2(\operatorname{arctg}\sqrt[5]{x^7})\cdot\sin(\operatorname{arctg}\sqrt[5]{x^7})\cdot\frac{\frac{7}{5}\sqrt[5]{x^2}}{1+\sqrt[5]{x^{14}}}\cdot\operatorname{tg}\frac{3}{x^4}\right.$
$\left.+\cos^3(\operatorname{arctg}\sqrt[5]{x^7})\cdot\frac{1}{\cos^2\frac{3}{x^4}}\cdot\frac{-12}{x^5}\right)$

o) $\left(e^{\ln(\operatorname{arctg}\frac{1}{x})\cdot\ln(2x)}\right)'=e^{\ln(\operatorname{arctg}\frac{1}{x})\cdot\ln(2x)}\cdot\left(\frac{1}{\operatorname{arctg}\frac{1}{x}}\cdot\frac{1}{1+\frac{1}{x^2}}\cdot\left(-\frac{1}{x^2}\right)\cdot\ln(2x)+\ln(\operatorname{arctg}\frac{1}{x})\cdot\frac{1}{2x}\cdot 2\right)$

p) $4\operatorname{tg}^3(\cos 7x)\cdot\frac{1}{\cos^2(\cos 7x)}\cdot(-7\sin 7x)\cdot\sin\frac{\operatorname{arctg}(e^x)}{x^3}+\operatorname{tg}^4(\cos 7x)\cdot\cos\frac{\operatorname{arctg}(e^x)}{x^3}\cdot$
$\frac{\frac{x^3e^x}{1+e^{2x}}-3x^2\operatorname{arctg}(e^x)}{x^6}$

q) $\frac{1}{\sqrt{\arcsin(e^{\sqrt[3]{x^2}})\cdot \operatorname{ctg}(\ln x)}} \cdot \left(\frac{1}{2\sqrt{\arcsin(e^{\sqrt[3]{x^2}})}} \cdot \frac{1}{\sqrt{1-e^{2\sqrt[3]{x^2}}}} \cdot e^{\sqrt[3]{x^2}} \cdot \frac{2}{3\sqrt[3]{x}} \cdot \operatorname{ctg}(\ln x) \right.$
$\left. + \sqrt{\arcsin(e^{\sqrt[3]{x^2}})} \cdot \frac{-1}{\sin^2(\ln x)} \frac{1}{x} \right)$

r) $\left(e^{\ln(\arcsin\frac{3}{x^3})\cdot\cos\sqrt[3]{x}}\right)' = e^{\ln(\arcsin\frac{3}{x^3})\cdot\cos\sqrt[3]{x}} \cdot \left(\frac{1}{\arcsin\frac{3}{x^3}} \cdot \frac{1}{\sqrt{1-\frac{9}{x^6}}} \cdot \frac{-9}{x^4} \cdot \cos\sqrt[3]{x} \right.$
$\left. + \ln(\arcsin\frac{3}{x^3}) \cdot (-\sin\sqrt[3]{x} \cdot \frac{1}{3\sqrt[3]{x^2}} \right)$