

Zestaw zadań do samodzielnej pracy nr 1
ME-DI, semestr zimowy, rok akademicki 2019/2020

1. Wyznaczyć asymptoty podanych funkcji:

a) $f(x) = \frac{\sin x}{x}$

b) $f(x) = \frac{x^3 - 1}{x - 1}$

c) $f(x) = \frac{1}{1 - x^2}$

d) $f(x) = e^{-x} \cdot \sin x + x$

e) $f(x) = \frac{x^3}{(1 + x^2)}$

f) $f(x) = \frac{1}{e^x - 1}$

g) $f(x) = \frac{1 - x^2}{x + 1}$

2. Zbadać monotoniczność oraz wyznaczyć ekstrema lokalne funkcji

a) $f(x) = 2x^3 - 15x^2 + 36x - 14$

b) $f(x) = 4x + \frac{1}{x}$

c) $f(x) = x^2 e^{-x}$

d) $f(x) = \frac{1}{x} + 5 \operatorname{arctg} x$

e) $f(x) = \frac{3x - 2}{x + 3}$

f) $f(x) = \frac{x^2 + 3}{x + 1}$

g) $f(x) = \frac{x^2 + 2x}{x^2 - 4}$

h) $f(x) = \frac{6}{x} + \frac{1}{2}x^2 + 5x$

i) $f(x) = 2xe^x - 8e^x - x^2 + 6x$

j) $f(x) = \ln(x + 1) - \operatorname{arctg} x$

k) $f(x) = \frac{x}{\ln x}$

3. Określić przedziały wypukłości oraz wyznaczyć punkty przegięcia podanych funkcji

a) $f(x) = x^4 - 6x^2 - 6x + 1$

b) $f(x) = \frac{x^2}{(x - 1)^3}$

c) $f(x) = xe^{-\frac{1}{2}x^2}$

d) $f(x) = e^{-x^3}$

- e) $f(x) = \frac{x+7}{x^2+5}$
- f) $f(x) = \frac{x^4}{x^3-1}$
- g) $f(x) = \frac{7x}{x^2+1}$
- h) $f(x) = \frac{x+1}{x^2+1}$
- i) $f(x) = \frac{x}{2+x^2}$
- j) $f(x) = x^2 + \ln(x+1)$

4. Obliczyć granicę, korzystając z twierdzenia de L'Hospitala

- a) $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+6x} - 1}{x} =$
- b) $\lim_{x \rightarrow 0^+} \frac{\ln \cos x}{\ln \cos 2x} =$
- c) $\lim_{x \rightarrow +\infty} \frac{\sqrt{6x-11}}{\ln x} =$
- d) $\lim_{x \rightarrow +\infty} x^2 e^{-x} =$
- e) $\lim_{x \rightarrow \frac{\pi}{4}} (\operatorname{tg} x)^{\operatorname{tg} 2x}$
- f) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x}\right)^{\sin x}$
- g) $\lim_{x \rightarrow \pi} \left(\frac{1}{\sin x} - \frac{1}{\pi - x}\right)$
- h) $\lim_{x \rightarrow 0} \frac{\sin 3x - 3x}{x^3} =$
- i) $\lim_{x \rightarrow 0} \frac{\cos 4x - 1}{7x^2}$
- j) $\lim_{x \rightarrow 0} \frac{2 \cos x - 2 + x^2}{x^4}$
- k) $\lim_{x \rightarrow 0} \frac{e^{2x} - e^{-2x} - 4x}{x - \sin x}$
- l) $\lim_{x \rightarrow 0} \frac{1+x}{x} =$
- m) $\lim_{x \rightarrow 0^+} \frac{\ln x}{\ln \sin x} =$
- n) $\lim_{x \rightarrow 1^-} \frac{2^x - 2^{2-x}}{(x-1)^2} =$
- o) $\lim_{x \rightarrow -\infty} \left[x \left(e^{\frac{1}{x}} - 1\right)\right] =$
- p) $\lim_{x \rightarrow \infty} \frac{\pi - 2 \operatorname{arctg} x}{\ln(x+1) - \ln x} =$
- q) $\lim_{x \rightarrow 0} \left(\frac{1}{x \sin x} - \frac{1}{x^2}\right) =$

- r) $\lim_{x \rightarrow 0^+} x^{\sin x} =$
s) $\lim_{x \rightarrow 1^-} (1-x)^{\cos \frac{\pi x}{2}} =$
t) $\lim_{x \rightarrow \infty} \left(\cos \frac{1}{x}\right)^x =$
u) $\lim_{x \rightarrow \frac{\pi}{2}^-} (\sin x)^{\operatorname{tg} x} =$
v) $(x+1)^{\frac{1}{\sqrt{x}}} =$

5. Obliczyć całki:

- a) $\int (5x^2 - 6x + 3 - \frac{2}{x} + \frac{5}{x^2}) dx =$, $\int (2x - 3x^2 + \frac{1}{2\sqrt{x}}) dx =$,
b) $\int (\sin x - \cos x) dx =$, $\int (e^{3x} + e^{\frac{x}{3}}) dx =$,
c) $\int \frac{dx}{\sin^2 x \cos^2 x} =$, $\int \operatorname{ctg}^2 x dx =$,
d) $\int \frac{x}{1+x^2} dx =$, $\int \frac{x^2}{a^3 + x^3} dx =$, $a \neq 0$,
e) $\int \frac{x\sqrt[3]{x} + \sqrt[4]{x}}{x^2} dx =$, $\int \frac{x}{\sqrt{x^2 - 6}} dx =$,
f) $\int \frac{x}{\sqrt{3 - 5x^2}} dx =$, $\int \frac{\cos x}{\sqrt{1 + \sin x}} dx =$
g) $\int \frac{e^x}{2e^x + 1} dx =$

6. Obliczyć całki metodą przez podstawienie:

- a) $\int (5x^3 - 3)^7 x^2 dx =$ $\int (x^2 + 4)^5 x dx =$ $\int \sin^3 x dx =$
b) $\int \frac{x}{(x^2 + 3)^6} dx =$ $\int \frac{x}{\sqrt{1 - x^4}} dx =$ $\int \frac{e^x}{e^{2x} + 1} dx =$,
c) $\int x e^{x^2} dx =$ $\int \frac{\operatorname{tg} x}{(1 + \operatorname{tg}^4 x) \cos^2 x} dx =$ $\int \frac{x^7}{\sqrt{1 - x^{16}}} dx =$,
d) $\int \frac{\sin^3 x}{\cos^{10} x} dx =$ $\int \frac{\ln x}{x} dx =$ $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx =$,
e) $\int \sqrt{3x + 1} dx =$ $\int \frac{x}{\sqrt[3]{2x^2 - 1}} dx =$ $\int \frac{x^2}{\cos^2 (x^3 + 1)} dx =$
f) $\int x \sqrt{1 + x^2} dx =$ $\int \frac{x^2}{\sqrt[5]{x^3 + 1}} dx =$ $\int \frac{1}{\sqrt{x} + \sqrt[3]{x}} dx =$,
g) $\int \cos x e^{\sin x} dx =$ $\int \frac{x^3}{\cos^2 x^4} dx =$ $\int \frac{\cos x}{\sin^2 x + 4} dx =$,
h) $\int \frac{(\ln x)^2}{x} dx =$ $\int \frac{1}{e^x + e^{-x}} dx =$ $\int \sin^5 x \cos x dx =$,
i) $\int \frac{\operatorname{tg} x}{\cos^2 x} dx =$ $\int \sin x \cos^2 x dx =$,

7. Obliczyć całki metodą całkowania przez części

- a) $\int \sqrt{x} \ln x dx =$ $\int \ln x dx =$ $\int \ln (x^2 + 1) dx =$,

$$\begin{aligned}
\text{b) } \int x^2 e^x dx &= \int x \cos x dx = \int x \cos 3x dx =, \\
\text{c) } \int \frac{\ln x}{x^5} dx &= \int e^{-2x} \sin 3x dx = \int e^x \cos x dx =, \\
\text{d) } \int \arcsin x dx &= \int x^2 e^{-x} dx = \int x \sin 2x dx =, \\
\text{e) } \int \sin \sqrt{x} dx &= \int x^3 e^{-x^2} dx = \int \frac{\ln x - 1}{\ln^2 x} dx =
\end{aligned}$$

8. Obliczyć:

$$\begin{aligned}
\text{a) } \int \frac{dx}{(3x-2)^4}, \\
\text{b) } \int \frac{dx}{(x-2)^2} \\
\text{c) } \int \frac{3x-4}{x^2-x-6} dx \\
\text{d) } \int \frac{2x-3}{x^2-3x+3} dx \\
\text{e) } \int \frac{x+13}{x^2-4x-5} dx \\
\text{f) } \int \frac{6x-13}{x^2-\frac{7}{2}x+\frac{3}{2}} dx \\
\text{g) } \int \frac{2x+6}{2x^2+3x+1} dx \\
\text{h) } \int \frac{\frac{5}{6}x-16}{x^2+3x-18} dx \\
\text{i) } \int \frac{5+x}{10x+x^2} dx \\
\text{j) } \int \frac{dx}{x^2+2x-1} \\
\text{k) } \int \frac{dx}{6x^2-13x+6} \\
\text{l) } \int \frac{7x}{4+5x^2} dx \\
\text{m) } \int \frac{2x-13}{(x-5)^2} dx \\
\text{n) } \int \frac{3x+1}{(x+2)^2} dx =
\end{aligned}$$

9. Obliczyć:

$$\begin{aligned}
\text{a) } \int \frac{dx}{2x^2-2x+5} C \\
\text{b) } \int \frac{dx}{3x^2+2x+1} \\
\text{c) } \int \frac{dx}{13-6x+x^2}
\end{aligned}$$

$$d) \int \frac{4x - 1}{2x^2 - 2x + 1} dx$$

$$e) \int \frac{2x - 1}{x^2 - 2x + 5} dx$$

$$f) \int \frac{dx}{9x^2 - 6x + 2}$$

$$g) \int \frac{2x - 20}{x^2 - 8x + 25} dx$$

$$h) \int \frac{3x + 4}{x^2 + 4x + 8} dx$$

$$i) \int \frac{x - 6}{x^2 - 3} dx$$

$$j) \int \frac{10x - 44}{x^2 - 4x + 20} dx$$

$$k) \int \frac{4x - 5}{x^2 - 6x + 10} dx$$

$$l) \int \frac{x + 6}{x^2 + 3} dx$$

10. Obliczyć:

$$a) \int \frac{5x}{2 + 3x} dx$$

$$b) \int \frac{2x + 3}{x - 1} dx,$$

$$c) \int \frac{x^2}{5x^2 + 12} dx$$

$$d) \int \frac{2x^2 + 7x + 20}{x^2 + 6x + 25} dx$$

$$e) \int \frac{x^3 - 4x^2 + 1}{(x - 2)^4} dx$$

$$\int \frac{2x + 1}{(x^2 + 1)^2} dx$$

$$f) \int \frac{2x}{(x^2 + 1)(x^2 + 3)} dx$$

$$g) \int \frac{dx}{x^3(x - 1)^2(x + 1)}$$

$$h) \int \frac{x^3 - 2x^2 + 7x + 4}{(x - 1)^2(x + 1)^2} dx$$

$$i) \int \frac{dx}{(x^2 + 4x + 8)^2}$$

$$j) \int \frac{5x^3 - 11x^2 + 5x + 4}{(x - 1)^4} dx$$

$$k) \int \frac{3x^2 + x - 2}{(x - 1)^3(x^2 + 1)} dx$$

11. Obliczyć:

- a) $\int \frac{x^3 + 2x - 6}{x^2 - x - 2} dx$
- b) $\int \frac{2x^3 - 19x^2 + 58x - 42}{x^2 - 8x + 16} dx$
- c) $\int \frac{x^4}{x^2 + 1} dx$
- d) $\int \frac{72x^6}{3x^2 + 2} dx$
- e) $\int \frac{2x^4 - 10x^3 + 21x^2 - 20x + 5}{x^2 - 3x + 2} dx$
- f) $\int \frac{x^2 + 2}{x + 2} dx$
- g) $\int \frac{x^3}{x^2 - 3x + 2} dx$
- h) $\int \frac{dx}{x^4 + 64}$
- i) $\int \frac{x^3 - 2x^2 + 5x - 8}{x^4 + 8x^2 + 16} dx$

12. Obliczyć:

- a) $\int \sqrt{2x + 1} dx,$
- b) $\int \frac{dx}{\sqrt{3 + 4x}}$
- c) $\int \frac{dx}{\sqrt[3]{3x - 4}}$
- d) $\int \frac{dx}{\sqrt[5]{(2x + 1)^3}}$
- e) $\int x \sqrt[3]{x - 4} dx$
- f) $\int x \sqrt[3]{3x - 1} dx$
- g) $\int \frac{x^2 + 1}{\sqrt{3x + 1}} dx$
- h) $\int \frac{x}{\sqrt[4]{2x + 3}} dx$
- i) $\int \frac{\sqrt{x + 1}}{x} dx$
- j) $\int \frac{dx}{\sqrt{x} + 2\sqrt[3]{x^2}}.$
- k) $\int \frac{dx}{\sqrt{x - 5} + \sqrt{x - 7}}$
- l) $\int \sqrt{\frac{1 - x}{1 + x}} \cdot \frac{1}{x} dx$

13. Obliczyć:

- a) $\int \frac{8x+3}{\sqrt{4x^2+3x+1}} dx$
- b) $\int \frac{dx}{\sqrt{2x-x^2}}$
- c) $\int \frac{dx}{\sqrt{1-9x^2}}$
- d) $\int \sqrt{1-4x^2} dx$
- e) $\int \frac{x+1}{\sqrt{8+2x-x^2}} dx$
- f) $\int \frac{dx}{\sqrt{x^2+3x+2}}$
- g) $\int \frac{3x-2}{\sqrt{4x^2-4x+5}} dx$
- h) $\int \sqrt{2x+x^2} dx$
- i) $\int \frac{x^2}{\sqrt{x^2+2x+2}} dx$
- j) $\int \frac{2x^2+3x+1}{\sqrt{x^2+1}} dx$
- k) $\int \frac{1}{x} \sqrt{\frac{2-x}{x}} dx$
- l) $\int \frac{dx}{\sqrt{x^2+4x+13-x}}$

14. Obliczyć:

- a) $\int \frac{dx}{\sqrt{4-2x-x^2}}$
- b) $\int \frac{(3x+1)}{\sqrt{x^2+5x-10}} dx$
- c) $\int \frac{2x+1}{\sqrt{2+x-3x^2}} dx$
- d) $\int \frac{10x-15}{\sqrt{36x^2-108x+77}} dx$
- e) $\int \frac{dx}{\sqrt{1-9x^2}}$
- f) $\int \frac{x-5}{\sqrt{5+4x-x^2}} dx C$
- g) $\int \frac{x+1}{\sqrt{8+2x-x^2}} dx$
- h) $\int \frac{dx}{\sqrt{4x^2+3x-1}}$
- i) $\int \frac{3x+2}{\sqrt{x^2-5x+19}} dx$

- j) $\int \frac{5x+2}{\sqrt{2x^2+8x-1}} dx$
 k) $\int \sqrt{x^2-4} dx$
 l) $\int \sqrt{x^2-3x+2} dx$
 m) $\int \frac{2x^2+3x+1}{\sqrt{x^2+1}} dx$
 n) $\int \frac{x^3-x+1}{\sqrt{x^2+2x+2}} dx$
 o) $\int x\sqrt{6+x-x^2} dx$
 p) $\int \frac{5x^2-2x+10}{\sqrt{3x^2-5x+8}} dx$
 q) $\int (3x-2)\sqrt{x^2-2x} dx$

Odpowiedzi

1. a) $y = 0$ -asymptota pozioma w $\pm\infty$, brak asymptot poziomych i ukośnych
 b) brak asymptot
 c) $x = 1, x = -1$ - asymptoty pionowe obustronne, $y = 0$ - asymptota pozioma w $\pm\infty$
 d) $y = x$ -asymptota ukośna w $+\infty$, brak asymptot pionowych i poziomych oraz ukośnej w $-\infty$
 e) $x = -1$ - asymptota pionowa obustronna, $y = x - 2$ - asymptota ukośna w $\pm\infty$
 f) $x = 0$ - asymptota pionowa obustronna, $y = -1$ - asymptota pozioma w $-\infty$, $y = 0$ - asymptota pozioma w $+\infty$
 g) $y = -x + 1$ - asymptota ukośna w $\pm\infty$, brak asymptot pionowych i poziomych
2. a) $\uparrow (-\infty, 2), (3, +\infty); \downarrow (2, 3); \max: x = 2, \min: x = 3$
 b) $\uparrow (-\infty, -\frac{1}{2}), (\frac{1}{2}, +\infty); \downarrow (-\frac{1}{2}, 0), (0, \frac{1}{2}); \max: x = -\frac{1}{2}, \min: x = \frac{1}{2}$
 c) $\uparrow (0, 2), \downarrow (-\infty, 0), (2, +\infty), \max: x = 2, \min: x = 0$
 d) $\uparrow (-\infty, -\frac{1}{2}), (\frac{1}{2}, +\infty); \downarrow (-\frac{1}{2}, 0), (0, \frac{1}{2}); \max: x = \frac{1}{2}, \min: x = -\frac{1}{2}$
 e) $\uparrow (-\infty, -3), (-3, +\infty)$ brak ekstremów;
 f) $\uparrow (-\infty, -3), (1, +\infty); \downarrow (-3, 1); \max: x = -3, \min: x = 1$
 g) $\downarrow (-\infty, -2), (-2, 2);$ brak ekstremów
 h) $\uparrow (-\infty, -3 - \sqrt{3}), (-3 + \sqrt{3}, 0), (0, 1); \downarrow (-3 - \sqrt{3}, -3 + \sqrt{3}), (1, +\infty); \max: x = -3 - \sqrt{3}, x = 1, \min: x = -3 + \sqrt{3}$
 i) $\uparrow (-\infty, 0), (3, +\infty); \downarrow (0, 3); \max: x = 0, \min: x = 3$
 j) $\uparrow (-1, 0), (1, +\infty); \downarrow (0, 1); \max: x = 0, \min: x = 1$
 k) $\uparrow (e, +\infty); \downarrow (0, 1), (1, e); \min: x = e$
3. a) punkty przegięcia: $P_1 = (-1, 2), P_2 = (1, -10)$

- b) punkty przegięcia: $P_1 = \left(-2 - \sqrt{3}, \frac{-7 - 4\sqrt{3}}{54 + 30\sqrt{3}}\right), P_2 = \left(-2 + \sqrt{3}, \frac{7 - 4\sqrt{3}}{15\sqrt{3} - 26}\right)$
- c) punkty przegięcia: $P_1 = (-\sqrt{3}, -\sqrt{3}e^{-\frac{3}{2}}), P_2 = (0, 0), P_3 = (\sqrt{3}, \sqrt{3}e^{-\frac{3}{2}})$
- d) punkty przegięcia: $P_1 = (0, 1), P_2 = (\sqrt[3]{\frac{2}{3}}, e^{-\frac{2}{3}})$
- e) punkty przegięcia: $P_1 = (3\sqrt{15} - 10, \frac{11+3\sqrt{15}}{20}), P_2 = (-1, 1), P_3 = (-3 - \sqrt{15} - 10, \frac{11-3\sqrt{15}}{20})$
- f) punkty przegięcia: $P_1 = (-\sqrt[3]{2}, -\frac{2\sqrt[3]{2}}{3}),$
- g) punkty przegięcia: $P_1 = (-\sqrt{3}, -\frac{7\sqrt{3}}{4}), P_2 = (0, 0), P_3 = (\sqrt{3}, \frac{7\sqrt{3}}{4})$
- h) punkty przegięcia: $P_1 = (-2 - \sqrt{3}, \frac{1-\sqrt{3}}{4}), P_2 = (\sqrt{3} - 2, \frac{1+\sqrt{3}}{4}), P_3 = (1, 1)$
- i) punkty przegięcia: $P_1 = (-\sqrt{6}, -\frac{\sqrt{6}}{8}), P_2 = (0, 0), P_3 = (\sqrt{6}, \frac{\sqrt{6}}{8})$
- j) punkty przegięcia: $P_1 = (\frac{\sqrt{2}}{2} - 1, \frac{3}{2} - \sqrt{2} - \frac{1}{2} \ln 2)$

4. a) 2
 b) $\frac{1}{4}$
 c) $+\infty$
 d) 0
 e) -1
 f) 1
 g) 0
 h) $-\frac{9}{2}$
 i) $-\frac{8}{7}$
 j) $\frac{1}{12}$
 k) 16
 l) 1
 m) 1
 n) $-\infty$
 o) 1
 p) 2
 q) $\frac{1}{6}$
 r) 1
 s) 1
 t) 1
 u) 1
 v) 1

5. $C \in \mathbb{R} :$

- a) $\frac{5}{3}x^3 - 3x^2 - 3x - \ln|x| + \frac{5}{x} + C,$ $x^2 - x^3 + \sqrt{x} + C,$
b) $-\cos x - \sin x + C,$ $\frac{1}{3}e^{3x} + 3e^{\frac{x}{3}} + C,$
c) $\operatorname{tg} x - \operatorname{ctg} x + C,$ $-\operatorname{ctg} x - x + C,$
d) $\frac{1}{2}\ln 1 + x^2 + C,$ $\frac{1}{3}\ln|a^3 + x^3| + C,$
e) $3x^{\frac{1}{3}} - \frac{4}{3}x^{-\frac{3}{4}} + C,$ $\sqrt{x^2 - 6} + C,$
f) $-\frac{1}{5}\sqrt{3 - 5x^2} + C,$ $2\sqrt{1 + \sin x} + C$
g) $\frac{1}{2}\ln(2e^x + 1) + C$

6. $C \in \mathbb{R} :$

- a) $\frac{1}{120}(5x^3 - 3)^8 + C$ $\frac{1}{12}(x^2 + 4)^6 + C$ $\frac{1}{3}\cos^3 x - \cos x + C$
b) $\frac{-1}{10(x^2 + 3)^5} + C$ $\frac{1}{2}\arcsin x^2 + C$ $\operatorname{arctg}(e^x) + C,$
c) $\frac{1}{2}e^{x^2} + C$ $\frac{1}{2}\operatorname{arctg}(\operatorname{tg}^2 x) + C$ $\frac{1}{8}\arcsin(x^8) + C,$
d) $\frac{1}{9\cos^9 x} - \frac{1}{7\cos^7 x} + C$ $\frac{1}{2}\ln^2 x + C$ $2\sin\sqrt{x} + C,$
e) $x \geq -\frac{1}{3} : \frac{2}{9}(3x + 1)^{\frac{3}{2}} + C,$ $\frac{3}{8}(2x^2 - 1)^{\frac{2}{3}}, x \neq \pm\frac{\sqrt{2}}{2},$
 $\frac{1}{3}\operatorname{tg}(x^3 + 1) + C, \cos(x^3 + 1) \neq 0$
f) $\frac{1}{3}\sqrt{(1 + x^2)^3} + C$ $\frac{5}{12}\sqrt[5]{(x^3 + 1)^4} + C$ $2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln|\sqrt[6]{x} + 1| + C,$
g) $e^{\sin x} + C$ $\frac{1}{4}\operatorname{tg} x^4, \cos x^4 \neq 0$ $\frac{1}{2}\operatorname{arctg}(\frac{1}{2}\sin x) + C,$
h) $x > 0, \frac{1}{3}\ln^3 x + C$ $\operatorname{arctg} e^x + C$ $\frac{1}{6}\sin^6 x + C,$
i) $\frac{1}{2}\operatorname{tg}^2 x + C, \cos x \neq 0$ $-\frac{1}{3}\cos^3 x + C,$

7. $C \in \mathbb{R}$

- a) $\frac{2}{3}x^{\frac{3}{2}}(\ln x - \frac{2}{3}) + C$ $x(\ln x - 1) + C$ $x \ln(x^2 + 1) - 2x + 2\operatorname{arctg} x + C,$
b) $(x^2 - 2x + 2)e^x + C$ $x \sin x + \cos x + C$ $\frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x + C,$
c) $-\frac{1}{4x^4}(\ln x + \frac{1}{4}) + C$ $-\frac{1}{13}e^{-2x}(2\sin 3x + 3\cos 3x) + C$ $\frac{1}{2}e^x(\sin x + \cos x) + C,$
d) $x \arcsin x + \sqrt{1 - x^2} + C$ $-e^{-x}(2x + x^2 + 2) + C$ $-\frac{1}{2}x \cos 2x + \frac{1}{4}\sin 2x + C,$
e) $-2\sqrt{x} \cos \sqrt{x} + 2\sin \sqrt{x} + C$ $\frac{e^{-x^2}(x^2 + 1)}{2} + C$ $\frac{x}{\ln x} + C$

8. a) $\int \frac{dx}{(3x - 2)^4} = \frac{-1}{9(3x - 2)^3} + C, x \neq \frac{2}{3}$

b) $\int \frac{dx}{(x - 2)^2} = \frac{-1}{x - 2} + C, x \neq 2$

c) $\int \frac{3x - 4}{x^2 - x - 6} dx = \ln|x - 3| + 2\ln|x + 2| + C, x \neq 3 \wedge x \neq -2$

d) $\int \frac{2x - 3}{x^2 - 3x + 3} dx = \ln|x^2 - 3x + 3| + C$

e) $\int \frac{x + 13}{x^2 - 4x - 5} dx = 3\ln|x - 5| - 2\ln|x + 1| + C, x \neq 5 \wedge x \neq -1$

- f) $\int \frac{6x-13}{x^2-\frac{7}{2}x+\frac{3}{2}}dx = 2\ln|x-3| + 4\ln|x-\frac{1}{2}| + C, x \neq 3 \wedge x \neq \frac{1}{2}$
- g) $\int \frac{2x+6}{2x^2+3x+1}dx = 5\ln|x+\frac{1}{2}| - 4\ln|x+1| + C, x \neq -\frac{1}{2} \wedge x \neq -1$
- h) $\int \frac{\frac{5}{6}x-16}{x^2+3x-18}dx = \frac{7}{3}\ln|x+6| - \frac{3}{2}\ln|x-3| + C, x \neq -6 \wedge x \neq 3$
- i) $\int \frac{5+x}{10x+x^2}dx = \frac{1}{2}\ln|10x+x^2| + C$
- j) $\int \frac{dx}{x^2+2x-1} = \frac{\sqrt{2}}{4}\ln\left|\frac{x+1-\sqrt{2}}{x+1+\sqrt{2}}\right| + C$
- k) $\int \frac{dx}{6x^2-13x+6} = \ln\left|\frac{2x-3}{3x-2}\right| + C, x \neq \frac{2}{3} \wedge x \neq \frac{3}{2}$
- l) $\int \frac{7x}{4+5x^2}dx = \frac{7}{10}\ln|4+5x^2| + C$
- m) $\int \frac{2x-13}{(x-5)^2}dx = 2\ln|x-5| + \frac{3}{x-5} + C, x \neq 5$
- n) $\int \frac{3x+1}{(x+2)^2}dx = 3\ln|x+2| + \frac{5}{x+2}, x \neq -2$

9. Obliczyć:

- a) $\int \frac{dx}{2x^2-2x+5} = \frac{1}{3}\arctg\frac{2x-1}{3} + C$
- b) $\int \frac{dx}{3x^2+2x+1} = \frac{\sqrt{2}}{2}\arctg\frac{3x+1}{\sqrt{2}} + C$
- c) $\int \frac{dx}{13-6x+x^2} = \frac{1}{2}\arctg\frac{x-3}{2} + C$
- d) $\int \frac{4x-1}{2x^2-2x+1}dx = \ln|2x^2-2x+1| + \arctg(2x-1) + C$
- e) $\int \frac{2x-1}{x^2-2x+5}dx = \ln|x^2-2x+5| + \frac{1}{2}\arctg\frac{x-1}{2} + C$
- f) $\int \frac{dx}{9x^2-6x+2} = \arctg(3x-1) + C$
- g) $\int \frac{2x-20}{x^2-8x+25}dx = \ln|x^2-8x+25| - 4\arctg\frac{x-4}{3} + C$
- h) $\int \frac{3x+4}{x^2+4x+8}dx = \frac{3}{2}\ln|x^2+4x+8| - \arctg\frac{x+2}{2} + C$
- i) $\int \frac{x-6}{x^2-3}dx = \frac{1}{2}\ln|x^2-3| + \sqrt{3}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + C$
- j) $\int \frac{10x-44}{x^2-4x+20}dx = 5\ln|x^2-4x+20| + 6\arctg\frac{x-2}{4} + C$
- k) $\int \frac{4x-5}{x^2-6x+10}dx = 2\ln|x^2-6x+10| + 7\arctg(x-3) + C$
- l) $\int \frac{x+6}{x^2+3}dx = \frac{1}{2}\ln|x^2+3| + 2\sqrt{3}\arctg\frac{x}{\sqrt{3}} + C$

10. Obliczyć:

- a) $\int \frac{5x}{2+3x} dx = \frac{5}{3}x - \frac{10}{9} \ln |2+3x| + C$
- b) $\int \frac{2x+3}{x-1} dx = 2x + 5 \ln |x-1| + C,$
- c) $\int \frac{x^2}{5x^2+12} dx = \frac{1}{5}x - \frac{\sqrt{60}}{25} \operatorname{arctg} \frac{\sqrt{5}x}{\sqrt{12}} + C$
- d) $\int \frac{2x^2+7x+20}{x^2+6x+25} dx = 2x - \frac{5}{2} \ln |x^2+6x+25| - \frac{15}{4} \operatorname{arctg} \frac{x+3}{4} + C$
- e) $\int \frac{x^3-4x^2+1}{(x-2)^4} dx = -\frac{29-30x+6x^2}{3(x-2)^3} + \ln |x-2| + C$
 $\int \frac{2x+1}{(x^2+1)^2} dx = \frac{1}{2} \operatorname{arctg} x + \frac{x-2}{2(x^2+1)} + C$
- f) $\int \frac{2x}{(x^2+1)(x^2+3)} dx = \frac{1}{2} \ln (x^2+1) - \frac{1}{2} \ln (x^2+3) + C$
- g) $\int \frac{dx}{x^3(x-1)^2(x+1)} = -\frac{1}{2x^2} - \frac{1}{x} + 2 \ln |x| - \frac{1}{4} \ln |x+1| - \frac{7}{4} \ln |x-1| + C$
- h) $\int \frac{x^3-2x^2+7x+4}{(x-1)^2(x+1)^2} dx = -\frac{5}{2(x-1)} - \ln |x-1| + \frac{3}{2(x+1)} + 2 \ln |x+1| + C$
- i) $\int \frac{dx}{(x^2+4x+8)^2} = \frac{1}{16} \cdot \frac{x+2}{(x^2+4x+8)^2} + \frac{3}{128} \cdot \frac{x+2}{x^2+4x+8} + \frac{3}{256} \operatorname{arctg} \frac{x+2}{2} + C$
- j) $\int \frac{5x^3-11x^2+5x+4}{(x-1)^4} dx = -\frac{1}{(x-1)^3} + \frac{1}{(x-1)^2} - \frac{4}{x-1} + 5 \ln |x-1| + C$
- k) $\int \frac{3x^2+x-2}{(x-1)^3(x^2+1)} dx = -\frac{1}{2(x-1)^2} - \frac{5}{2(x-1)} - \frac{3}{2} \ln |x-1| + \frac{3}{4} \ln |x^2+1| - \operatorname{arctg} x + C$

11. Obliczyć:

- a) $\int \frac{x^3+2x-6}{x^2-x-2} dx = \frac{1}{2}x^2 + x + 3 \ln |x+1| + 2 \ln |x-2| + C$
- b) $\int \frac{2x^3-19x^2+58x-42}{x^2-8x+16} dx = x^2 - 3x - \frac{14}{x-4} + 2 \ln |x-4| + C, x \neq 4$
- c) $\int \frac{x^4}{x^2+1} dx = \frac{1}{3}x^3 - x + \operatorname{arctg} x + C$
- d) $\int \frac{72x^6}{3x^2+2} dx = \frac{9}{5}x^5 - 2x^3 + 4x - \frac{4\sqrt{6}}{3} \operatorname{arctg} \frac{\sqrt{3}x}{\sqrt{2}} + C$
- e) $\int \frac{2x^4-10x^3+21x^2-20x+5}{x^2-3x+2} dx = \frac{2}{3}x^3 - 2x^2 + 5x + \ln |x-2| + 2 \ln |x-1| + C$
- f) $\int \frac{x^2+2}{x+2} dx = \frac{1}{2}x^2 - 2x + 6 \ln |x+2| + C$
- g) $\int \frac{x^3}{x^2-3x+2} dx = \frac{1}{2}x^2 + 3x - \ln |x-1| + 8 \ln |x-2| + C$
- h) $\int \frac{dx}{x^4+64} = \frac{1}{64} \left(\frac{1}{2} \ln \frac{x^2+4x+8}{x^2-4x+8} + \operatorname{arctg} \frac{x+2}{2} - \operatorname{arctg} \frac{x-2}{2} \right) + C$

$$\text{i)} \int \frac{x^3 - 2x^2 + 5x - 8}{x^4 + 8x^2 + 16} dx = -\frac{1}{2(x-1)^2} - \frac{5}{2(x-1)} - \frac{3}{2} \ln|x-1| + \frac{3}{4} \ln|x^2+1| - \arctg x + C$$

12. Obliczyć:

$$\text{a)} \int \sqrt{2x+1} dx = \frac{1}{3}(2x+1)^{\frac{3}{2}} + C, x \geq -\frac{1}{2},$$

$$\text{b)} \int \frac{dx}{\sqrt{3+4x}} = \frac{1}{2}\sqrt{3+4x} + C, x > -\frac{3}{4}$$

$$\text{c)} \int \frac{dx}{\sqrt[3]{3x-4}} = \frac{1}{2}(3x-4)^{\frac{2}{3}} + C, x \neq \frac{4}{3}$$

$$\text{d)} \int \frac{dx}{\sqrt[5]{(2x+1)^3}} = \frac{5}{4}(2x+1)^{\frac{2}{5}} + C, x \neq -\frac{1}{2}$$

$$\text{e)} \int x\sqrt[3]{x-4} dx = \frac{3}{7}(x^2-x-12)\sqrt[3]{x-4} + C$$

$$\text{f)} \int x\sqrt[3]{3x-1} dx = \frac{1}{28}(12x^2-x-1)\sqrt[3]{3x-1} + C$$

$$\text{g)} \int \frac{x^2+1}{\sqrt{3x+1}} dx = \frac{2}{405}(27x^2-12x+143)\sqrt{3x+1} + C$$

$$\text{h)} \int \frac{x}{\sqrt[4]{2x+3}} dx = \frac{2}{7}(x-2)(2x+3)^{\frac{3}{4}} + C, x > -\frac{3}{2}$$

$$\text{i)} \int \frac{\sqrt{x+1}}{x} dx = 2\sqrt{x+1} + \ln \left| \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1} \right| + C$$

$$\text{j)} \int \frac{dx}{\sqrt{x}+2\sqrt[3]{x^2}} = \frac{3}{2} \left(\sqrt[3]{x} - \sqrt[6]{x} + \frac{1}{2} \ln |2\sqrt[6]{x}+1| \right).$$

$$\text{k)} \int \frac{dx}{\sqrt{x-5}+\sqrt{x-7}} = \frac{1}{3} \left((x-5)^{\frac{3}{2}} - (x-7)^{\frac{3}{2}} \right) + C, x \geq 7$$

$$\text{l)} \int \sqrt{\frac{1-x}{1+x}} \cdot \frac{1}{x} dx = \ln |1 - \sqrt{1-x^2}| - \ln|x| - \arcsin x + C, x \in (-1, 0) \cup (0, 1)$$

13. Obliczyć:

$$\text{a)} \int \frac{8x+3}{\sqrt{4x^2+3x+1}} dx = 2\sqrt{4x^2+3x+1} + C$$

$$\text{b)} \int \frac{dx}{\sqrt{2x-x^2}} = \arcsin \frac{x-1}{\sqrt{2}} + C$$

$$\text{c)} \int \frac{dx}{\sqrt{1-9x^2}} = \frac{1}{3} \arcsin 3x + C$$

$$\text{d)} \int \sqrt{1-4x^2} dx = \frac{1}{2}x\sqrt{1-4x^2} + \frac{1}{4} \arcsin x + C, x \in \left\langle -\frac{1}{2}, \frac{1}{2} \right\rangle$$

$$\text{e)} \int \frac{x+1}{\sqrt{8+2x-x^2}} dx = -\sqrt{8+2x-x^2} + 2 \arcsin \left(\frac{x-1}{3} \right) + C$$

$$\text{f)} \int \frac{dx}{\sqrt{x^2+3x+2}} = \ln \left| x + \frac{3}{2} + \sqrt{x^2+3x+2} \right| + C, x \in (-\infty, -2) \cup (-1, +\infty)$$

$$\text{g)} \int \frac{3x-2}{\sqrt{4x^2-4x+5}} dx = \frac{3}{4}\sqrt{4x^2-4x+5} - \frac{1}{4} \ln |2x-1 + \sqrt{4x^2-4x+5}| + C$$

- h) $\int \sqrt{2x+x^2}dx = \frac{1}{2}(x+1)\sqrt{2x+x^2} - \frac{1}{2}\ln|1+x+\sqrt{2x+x^2}| + C$
- i) $\int \frac{x^2}{\sqrt{x^2+2x+2}}dx = \frac{1}{2}(x-3)\sqrt{x^2+2x+2} + \frac{1}{2}\ln|x+1+\sqrt{x^2+2x+2}| + C$
- j) $\int \frac{2x^2+3x+1}{\sqrt{x^2+1}}dx = (x+3)\sqrt{x^2+1} + C$
- k) $\int \frac{1}{x}\sqrt{\frac{2-x}{x}}dx = -2\sqrt{\frac{2-x}{x}} + 2\operatorname{arctg}\sqrt{\frac{2-x}{x}} + C$
- l) $\int \frac{dx}{\sqrt{x^2+4x+13}-x} = -\frac{13}{8}\ln|\sqrt{x^2+4x+13}-x| + \frac{9}{8}\ln|2+x-\sqrt{x^2+4x+13}| - \frac{9}{4(2+x-\sqrt{x^2+4x+13})} + C$

14. Obliczyć:

- a) $\int \frac{dx}{\sqrt{4-2x-x^2}} = \arcsin \frac{x+1}{\sqrt{5}} + C$
- b) $\int \frac{(3x+1)}{\sqrt{x^2+5x-10}}dx = 3\sqrt{x^2+5x-10} - \frac{13}{2}\ln\left|x+\frac{5}{2}+\sqrt{x^2+5x-10}\right| + C$
- c) $\int \frac{2x+1}{\sqrt{2+x-3x^2}}dx = -\frac{2}{3}\sqrt{2+x-3x^2} + \frac{4\sqrt{3}}{9}\arcsin \frac{6x-1}{5} + C$
- d) $\int \frac{10x-15}{\sqrt{36x^2-108x+77}}dx = \frac{5}{18}\sqrt{36x^2-108x+77} + C$
- e) $\int \frac{dx}{\sqrt{1-9x^2}} = \frac{1}{3}\arcsin 3x + C$
- f) $\int \frac{x-5}{\sqrt{5+4x-x^2}}dx = -\sqrt{5+4x-x^2} - 3\arcsin \frac{x-2}{3} + C$
- g) $\int \frac{x+1}{\sqrt{8+2x-x^2}}dx = -\sqrt{8+2x-x^2} + 2\arcsin \frac{x-1}{3} + C$
- h) $\int \frac{dx}{\sqrt{4x^2+3x-1}} = \frac{1}{2}\ln\left|2x+\frac{3}{4}+\sqrt{4x^2+3x-1}\right| + C$
- i) $\int \frac{3x+2}{\sqrt{x^2-5x+19}}dx = 3\sqrt{x^2-5x+19} + \frac{19}{2}\ln\left|x-\frac{5}{2}+\sqrt{x^2-5x+19}\right| + C$
- j) $\int \frac{5x+2}{\sqrt{2x^2+8x-1}}dx = \frac{5}{2}\sqrt{2x^2+8x-1} - 4\sqrt{2}\ln\left|x+2+\sqrt{x^2+4x-\frac{1}{2}}\right| + C$
- k) $\int \sqrt{x^2-4}dx = \frac{1}{2}x\sqrt{x^2-4} - 2\ln|x+\sqrt{x^2-4}| + C$
- l) $\int \sqrt{x^2-3x+2}dx = \frac{1}{2}\left(x-\frac{3}{2}\right)\sqrt{x^2-3x+2} - \frac{1}{8}\ln\left|x-\frac{3}{2}+\sqrt{x^2-3x+2}\right| + C$
- m) $\int \frac{2x^2+3x+1}{\sqrt{x^2+1}}dx = (x+3)\sqrt{x^2+1} + C$
- n) $\int \frac{x^3-x+1}{\sqrt{x^2+2x+2}}dx = \frac{1}{6}(2x^2-5x+6)\sqrt{x^2+2x+2} + \frac{5}{2}\ln\left|x+1+\sqrt{x^2+2x+2}\right| + C$
- o) $\int x\sqrt{6+x-x^2}dx = \frac{1}{24}(8x^2-2x-51)\sqrt{6+x-x^2} - \frac{25}{16}\arcsin \frac{1-2x}{5} + C$

$$\text{p) } \int_C \frac{5x^2 - 2x + 10}{\sqrt{3x^2 - 5x + 8}} dx = \left(\frac{5}{6}x + \frac{17}{12}\right) \sqrt{3x^2 - 5x + 8} + \frac{55}{24}\sqrt{3} \ln \left| 3x - \frac{5}{2} + \sqrt{3x^2 - 5x + 8} \right| +$$

$$\text{q) } \int (3x-2)\sqrt{x^2-2x}dx = \left(x^2 - \frac{3}{2}x - \frac{1}{2}\right) \sqrt{x^2-2x} - \frac{1}{2} \ln |x-1 + \sqrt{x^2-2x}| + C$$