

Zestaw zadań do samodzielnej pracy nr 2 - Pochodne
ME-DI, semestr zimowy, rok akademicki 2019/2020

1. Obliczyć pochodne funkcji:

a) $f(x) = 3\sqrt[3]{x^7} - 14\sqrt[4]{x^{13}} + \frac{4}{7\sqrt{x}} + \sqrt{7^3}, \quad g(x) = \frac{2-x^2}{\sqrt{2+\sqrt{3}}}$

b) $f(x) = x^3 \cos x, \quad g(x) = \sin x \cos x, \quad h(x) = x \ln x$

c) $f(x) = (5x^7 - 2x^3 + x - 20)e^x, \quad g(x) = (9x^7 + 3x^{-5} - 3x^{-11})2^x,$
 $h(x) = x \arcsin x$

d) $f(x) = \frac{4x^7 + 3x^5 - 2x^4 + 7x - 2}{3x^4}, \quad g(x) = \frac{2-x^2}{2x^3+x+3}, \quad h(x) = \frac{8x^3}{x^3+x-1}$

e) $f(x) = \frac{3x^2}{7x^5-x+2}, \quad g(x) = \frac{x^2-2x+3}{x^2+2x-3}, \quad h(x) = \frac{5}{2x^2-5x+1}$

f) $f(x) = \sqrt{3x^2-7x+12}, \quad g(x) = (4x^5-7x^3+14x^2-5)^3$

g) $y = \sin 4x, \quad s = (3t+1)^7, \quad v = (4z^2-5z+13)^{\frac{1}{3}}$

h) $f(x) = \cos^3 x, \quad g(x) = \operatorname{tg}^4 x, \quad h(x) = \arcsin \frac{2}{x}$

i) $f(x) = e^{-x}, \quad g(x) = e^{4x^3-6x+1}, \quad h(x) = e^{\sin x}$

j) $f(x) = 7 \cdot 5^{10x}, \quad g(x) = 5 \cdot 10^{3x}, \quad h(x) = 3^x \cdot x^3$

k) $f(x) = 10 \ln x + e^x, \quad g(x) = \ln 3x, \quad h(x) = 5 \ln 10x$

l) $f(x) = 3 \ln \frac{5}{x-2}, \quad g(x) = \ln \sin x, \quad h(x) = \log_3 x$

m) $f(x) = \log_5 (x^2-1), \quad g(x) = \log_4 (36-x^2), \quad h(x) = \sqrt{\ln x}$

n) $f(x) = x^x, \quad g(x) = x^{\sin x}, \quad h(x) = x^{5x}, \text{ dla } x > 0$

2. Obliczyć pochodne:

a) $\left[\ln \left(\frac{x^2+2}{x^4+7} \right) \right]' =$

b) $\left[\left(\frac{x^2+2x+1}{x^3+27} \right)^3 \right]' =$

c) $\left[\left(3 + \frac{1}{x^2} \right) \cdot \ln \sqrt{x} + e^3 \right]' =$

d) $\left[\left(\sqrt{x} + \frac{2}{x} \right) \cdot \ln (x^2+4) + \sqrt[3]{2} \right]' =$

e) $\left[\left(6\sqrt[3]{x} - \frac{3}{x^3} \right) \cdot e^{x^2+5x+4} + \sqrt{2} \right]' =$

f) $\left[\left(3x^2 - \frac{1}{x} + \sqrt{x} \right) \cdot e^{\sin x+5} + e^3 \right]' =$

g) $\left(\sqrt{\ln \left[\arcsin \frac{x^2+5x+7}{x^2+1} \right]} \right)' =$

h) $\left(\sqrt[3]{1 + \frac{5}{\operatorname{tg} x}} \cdot \arcsin \frac{1}{x} \right)' =$

- i) $\left(e^{\sin^3 x + 7} \cdot \cos \sqrt{x^2 + 1} + \sqrt{7}\right)' =$
- j) $\left(\sin \left[\ln \sqrt{\frac{x^3 + 4x^2 + 1}{x^2 + 2}} \right]\right)' =$
- k) $\left(\arctg \sqrt[3]{\operatorname{tg} x} \cdot e^{\arcsin(\frac{1}{x})} + \sqrt{2}\right)' =$
- l) $\left(\ln \left[\frac{\cos^2 x + 1}{\sin x} \right] \cdot \cos \sqrt{\frac{3}{x}}\right)' =$
- m) $\left(e^{\sqrt{\arcsin \frac{1}{x}}} \cdot \ln \frac{\sin(4x^2 + 5x + 2)}{\operatorname{ctg} x}\right)' =$
- n) $\left(\left[\cos^3(\arctg \sqrt[5]{x^7}) \cdot \operatorname{tg} \frac{3}{x^4} + \sqrt{2}\right]^3\right)' =$
- o) $\left(\left(\arctg \frac{1}{x}\right)^{\ln(2x)}\right)' =$
- p) $\left(\operatorname{tg}^4(\cos 7x) \cdot \sin \frac{\arctg(e^x)}{x^3} + \sqrt{3}\right)' =$
- q) $\left(\ln \left[\sqrt{\arcsin(e^{\sqrt[3]{x^2}})} \cdot \operatorname{ctg}(\ln x) \right]\right)' =$
- r) $\left(\left(\arcsin \frac{3}{x^3}\right)^{\cos \sqrt[3]{x}}\right)' =$

Odpowiedzi

1. a) $f'(x) = 7x^{\frac{4}{3}} - \frac{91}{2}x^{\frac{9}{4}} - \frac{2}{7}x^{-\frac{3}{2}}, x \neq 0 \quad g'(x) = \frac{-2x}{\sqrt{2 + \sqrt{3}}}$
- b) $f'(x) = 3x^2 \cos x - x^3 \sin x, \quad g'(x) = \cos^2 x - \sin^2 x, \quad h'(x) = \ln x + 1, x > 0$
- c) $f'(x) = (5x^7 + 35x^6 - 2x^3 - 6x^2 + x - 19)e^x,$
 $g'(x) = 2^x [(9x^7 + 3x^{-5} - 3x^{-11}) \ln 2 + 63x^6 - 15x^{-6} + 33x^{-12}], x \neq 0$
 $h'(x) = \arcsin x + \frac{x}{\sqrt{1 - x^2}}, x \in (-1, 1)$
- d) $f'(x) = \frac{12x^7 + 3x^5 - 21x + 8}{3x^5}, x \neq 0$
 $g'(x) = \frac{2x^4 - 13x^2 - 6x - 2}{(2x^3 + x + 3)^2}, 2x^3 + x + 3 \neq 0$
 $h'(x) = \frac{16x^3 - 24x^2}{(x^3 + x - 1)^2}, x^3 + x - 1 \neq 0$
- e) $f'(x) = \frac{-63x^5 - 3x^2 + 12x}{(7x^5 - x + 2)^2}, 7x^5 - x + 2 \neq 0$
 $g'(x) = \frac{4x^2 - 12x}{(x^2 + 2x - 3)^2}, x \neq -3 \wedge x \neq 1$
 $h'(x) = \frac{-20x + 25}{(2x^2 - 5x + 1)^2}, 2x^2 - 5x + 1 \neq 0$
- f) $f'(x) = \frac{6x - 7}{2\sqrt{3x^2 - 7x + 12}}$
 $g'(x) = 3(4x^5 - 7x^3 + 14x^2 - 5)^2(20x^4 - 21x^2 + 28x)$

- g) $y' = 4 \cos 4x, \quad s' = 21(3t+1)^6, \quad v' = \frac{8z-5}{3\sqrt[3]{(4z^2-5z+13)^2}}$
- h) $f'(x) = -3 \cos^2 x \sin x$
 $g'(x) = 4 \operatorname{tg}^3 x \cdot \frac{1}{\cos^2 x} = \frac{4 \sin^3 x}{\cos^5 x}, x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$
 $h'(x) = \frac{-2}{z\sqrt{1-\frac{4}{x^2}}}, x \in (-\infty, -2) \cup (2, +\infty)$
- i) $f'(x) = -e^{-x}, \quad g'(x) = (12x^2 - 6)e^{4x^3-6x+1}, \quad h'(x) = \cos x \cdot e^{\sin x}$
- j) $f'(x) = 70 \ln 5 \cdot 5^{10x}, \quad g'(x) = 15 \ln 10 \cdot 10^{3x}, \quad h'(x) = 3^x \cdot x^2(x \ln 3 + 3)$
- k) $f'(x) = \frac{10}{x} + e^x, x \neq 0 \quad g'(x) = \frac{1}{x}, x \neq 0, \quad h'(x) = \frac{5}{x}, x \neq 0$
- l) $f'(x) = \frac{-3}{x-2}, x \neq 2, \quad g'(x) = \operatorname{ctg} x, \quad h'(x) = \frac{1}{x \ln 3}, x \neq 0$
- m) $f'(x) = \frac{2x}{\ln 5(x^2-1)}, x \neq -1 \wedge x \neq 1$
 $g'(x) = \frac{-2x}{\ln 4(36-x^2)}, x \neq -6 \wedge x \neq 6$
 $h'(x) = \frac{1}{2x\sqrt{\ln x}}, x > 1$
- n) $f'(x) = x^x(\ln x + 1), x > 0$
 $g'(x) = x^{\sin x} \left(\frac{\sin x}{x} + \cos x \ln x \right), x > 0$
 $h'(x) = 5x^{5x}(\ln x + 1), x > 0$
2. a) $\frac{-2x^5 - 8x^3 + 14x}{(x^4 + 7)(x^2 + 2)}$
- b) $3 \left(\frac{x^2 + 2x + 1}{x^3 + 27} \right) \cdot \frac{-x^4 - 4x^3 - 3x^2 + 54x + 54}{(x^3 + 27)^2}$
- c) $-\frac{2}{x^3} \cdot \ln \sqrt{x} + \left(3 + \frac{1}{x^2} \right) \cdot \frac{1}{2x}$
- d) $\left(\frac{1}{2\sqrt{x}} - \frac{2}{x^2} \right) \cdot \ln(x^2 + 4) + \left(\sqrt{x} + \frac{2}{x} \right) \cdot \frac{2x}{x^2 + 4}$
- e) $\left(\frac{2}{\sqrt[3]{x^2}} + \frac{9}{x^4} \right) \cdot e^{x^2+5x+4} + \left(6\sqrt[3]{x} - \frac{3}{x^3} \right) e^{x^2+5x+4} \cdot (2x + 5)$
- f) $\left(6x + \frac{1}{x^2} + \frac{1}{2\sqrt{x}} \right) \cdot e^{\sin x+5} + \left(3x^2 - \frac{1}{x} + \sqrt{x} \right) \cdot e^{\sin x+5} \cdot \cos x$
- g) $\frac{1}{2\sqrt{\ln \left(\arcsin \frac{x^2+5x+7}{x^2+1} \right)}} \cdot \frac{1}{\arcsin \frac{x^2+5x+7}{x^2+1}} \cdot \frac{1}{\sqrt{1 - \left(\frac{x^2+5x+7}{x^2+1} \right)^2}}$
 $\cdot \frac{(2x+5)(x^2+1) - 2x(x^2+5x+7)}{(x^2+1)^2}$
- h) $\frac{1}{3\sqrt[3]{\left(1 + \frac{5}{\operatorname{tg} x} \right)^2}} \cdot \frac{-5}{\operatorname{tg}^2 x} \cdot \frac{1}{\cos^2 x} \cdot \operatorname{arc} \operatorname{tg}^2 \frac{1}{x} + \sqrt[3]{1 + \frac{5}{\operatorname{tg} x}} \cdot 2 \operatorname{arc} \operatorname{tg} \frac{1}{x} \cdot \frac{1}{1 + \left(\frac{1}{x} \right)^2} \cdot$

- $\left(-\frac{1}{x^2}\right)$
 i) $e^{\sin^3 x + 7} \cdot (3 \sin^2 x \cos x) \cdot \cos \sqrt{x^2 + 1} + e^{\sin^3 x + 7} \cdot (-\sin \sqrt{x^2 + 1}) \cdot \frac{2x}{2\sqrt{x^2 + 1}}$
 j) $\cos \left(\ln \sqrt{\frac{x^3 + 4x^2 + 1}{x^2 + 2}} \right) \cdot \frac{1}{\sqrt{\frac{x^3 + 4x^2 + 1}{x^2 + 2}}} \cdot \frac{1}{2\sqrt{\frac{x^3 + 4x^2 + 1}{x^2 + 2}}} \cdot \frac{(3x^2 + 8x)(x^2 + 2) - 2x(x^3 + 4x^2 + 1)}{(x^2 + 2)^2}$
 k) $\frac{1}{1 + (\sqrt[3]{\operatorname{tg} x})^2} \cdot \frac{1}{3\sqrt[3]{\operatorname{tg}^2 x}} \cdot \frac{1}{\cos^2 x} \cdot e^{\arcsin \frac{1}{x}} + \arcsin \left(\sqrt[3]{\operatorname{tg} x} \right) \cdot e^{\arcsin \frac{1}{x}} \cdot \frac{1}{\sqrt{1 - \left(\frac{1}{x}\right)^2}} \cdot \left(-\frac{1}{x^2}\right)$
 l) $\frac{\sin x}{\cos^2 x + 1} \cdot \frac{-2 \cos x \sin^2 x - \cos x (\cos^2 x + 1)}{\sin^2 x} \cdot \cos \sqrt{\frac{3}{x}} + \ln \left(\frac{\cos^2 x + 1}{\sin x} \right) \cdot \left(-\sin \sqrt{\frac{3}{x}} \right) \cdot \frac{1}{2\sqrt{\frac{3}{x}}} \cdot \left(-\frac{3}{x^2}\right)$
 m) $e^{\sqrt{\arcsin \frac{1}{x}}} \cdot \frac{1}{2\sqrt{\arcsin \frac{1}{x}}} \cdot \frac{1}{\sqrt{1 - \frac{1}{x^2}}} \cdot \ln \frac{\sin(4x^2 + 5x + 2)}{\operatorname{ctg} x} + e^{\sqrt{\arcsin \frac{1}{x}}} \cdot \frac{\operatorname{ctg} x}{\sin(4x^2 + 5x + 2)} \cdot \frac{(8x + 5) \cos(4x^2 + 5x + 2) \cdot \operatorname{ctg} x - \sin(4x^2 + 5x + 2) \cdot \frac{-1}{\sin^2 x}}{\operatorname{ctg}^2 x}$
 n) $3 \left(\cos^3(\operatorname{arctg} \sqrt[5]{x^7}) \cdot \operatorname{tg} \frac{3}{x^4} + \sqrt{2} \right)^2 \cdot \left(3 \cos^2(\operatorname{arctg} \sqrt[5]{x^7}) \cdot \sin(\operatorname{arctg} \sqrt[5]{x^7}) \cdot \frac{\frac{7}{5} \sqrt[5]{x^2}}{1 + \sqrt[5]{x^{14}}} \cdot \operatorname{tg} \frac{3}{x^4} + \cos^3(\operatorname{arctg} \sqrt[5]{x^7}) \cdot \frac{1}{\cos^2 \frac{3}{x^4}} \cdot \frac{-12}{x^5} \right)$
 o) $\left(e^{\ln(\operatorname{arctg} \frac{1}{x}) \cdot \ln(2x)} \right)' = e^{\ln(\operatorname{arctg} \frac{1}{x}) \cdot \ln(2x)} \cdot \left(\frac{1}{\operatorname{arctg} \frac{1}{x}} \cdot \frac{1}{1 + \frac{1}{x^2}} \cdot \left(-\frac{1}{x^2}\right) \cdot \ln(2x) + \ln(\operatorname{arctg} \frac{1}{x}) \cdot \frac{1}{2x} \cdot 2 \right)$
 p) $4 \operatorname{tg}^3(\cos 7x) \cdot \frac{1}{\cos^2(\cos 7x)} \cdot (-7 \sin 7x) \cdot \sin \frac{\operatorname{arctg}(e^x)}{x^3} + \operatorname{tg}^4(\cos 7x) \cdot \cos \frac{\operatorname{arctg}(e^x)}{x^3} \cdot \frac{\frac{x^3 e^x}{1 + e^{2x}} - 3x^2 \operatorname{arctg}(e^x)}{x^6}$
 q) $\frac{1}{\sqrt{\arcsin(e^{\sqrt[3]{x^2}}) \cdot \operatorname{ctg}(\ln x)}} \cdot \left(\frac{1}{2\sqrt{\arcsin(e^{\sqrt[3]{x^2}})}} \cdot \frac{1}{\sqrt{1 - e^{2\sqrt[3]{x^2}}}} \cdot e^{\sqrt[3]{x^2}} \cdot \frac{2}{3\sqrt[3]{x}} \cdot \operatorname{ctg}(\ln x) + \sqrt{\arcsin(e^{\sqrt[3]{x^2}})} \cdot \frac{-1}{\sin^2(\ln x)} \cdot \frac{1}{x} \right)$
 r) $\left(e^{\ln(\arcsin \frac{3}{x^3}) \cdot \cos \sqrt[3]{x}} \right)' = e^{\ln(\arcsin \frac{3}{x^3}) \cdot \cos \sqrt[3]{x}} \cdot \left(\frac{1}{\arcsin \frac{3}{x^3}} \cdot \frac{1}{\sqrt{1 - \frac{9}{x^6}}} \cdot \frac{-9}{x^4} \cdot \cos \sqrt[3]{x} + \ln(\arcsin \frac{3}{x^3}) \cdot (-\sin \sqrt[3]{x} \cdot \frac{1}{3\sqrt[3]{x^2}}) \right)$